Symmetries in Geometric Engineering

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with B. Acharya, F. Baume, N. Braeger, V. Chakrabhavi, M. Cvetič, M. Del Zotto, R. Donagi, J. J. Heckman, C. Murdia, E. Torres, A. Turner, X. Yu, H. Y. Zhang

CGP Series, Wednesday October 9th, 2024

Motivation: Recent Advances in QFT

- In well-known theories such as QED_4 new symmetries were discovered [Choi, Lam, Shao; 2022]

$$d * j_A = \frac{1}{8\pi^2} F \wedge F$$

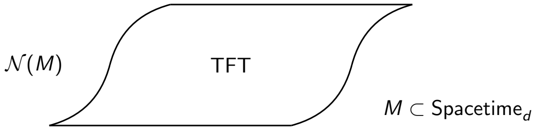
- Such categorical symmetries were also noticed throughout many theories [Kaidi, Ohmori, Zheng; 2021 & 2022], [Bhardwaj, Schäfer-Nameki, Wu; 2022], [Bartsch, Bullimore, Ferrari, Pearson; 2022], [Heckman, Hübner, Torres, Zhang; 2022], [Vladimir Bashmakov, Del Zotto, Hasan, Kaidi; 2022], [Cordova, Hong, Ohmori; 2022], [Cordova, Koren; 2022]

$$\text{Symmetry} \leftrightarrow \text{Topological Operator}$$
$$\mathcal{N} \otimes \mathcal{N}^\dagger \neq 1$$

- $\rightarrow$ What are the topological operators of a given QFT? [Gaiotto, Kapustin, Seiberg, Willett; 2014]

- Topological Operators $\mathcal{N}$ support TFTs

  [Kaidi, Ohmori, Zheng; 2021], [Choi, Cordova, Lam, Shao; 2021]



$\rightarrow$ "$\mathcal{N}$ is a Topological Brane within the ambient QFT_d"

- Streamlined construction of topological operators? Non-Lagrangian methods?

- How to characterize the topological operators of a QFT_d?
  ($\rightarrow$ Symmetry Theory_D in dimension $D > d$) [Gaiotto, Kulp; 2021], [Apruzzi, Bonetti, García Etxebarria, Hosseini, Schäfer-Nameki; 2021], [Freed, Moore, Teleman; 2022]

Motivation: String-constructed QFTs

- SCFT_D in dimension $D = 5, 6$ are strongly coupled and non-Lagrangian, easily constructed in string theory, and progenitor theories for many lower-dimensional QFTs

  [Heckman, Morrison, Vafa; 2014], [Heckman, Morrison, Rudelius, Vafa; 2015], [Apruzzi, Lawrie, Lin, Schäfer-Nameki, Wang; 2019], [Closset, Del Zotto; 2020], ...

- Consider string backgrounds $\mathbb{R}^{1,d-1} \times X$ constructing QFTs with spacetime $\mathbb{R}^{1,d-1}$ and symmetries $\mathrm{Sym}(\mathrm{QFT}_X)$

$$X \xrightarrow{\text{String Theory}} \mathrm{QFT}_X \longrightarrow \mathrm{Sym}(\mathrm{QFT}_X)$$

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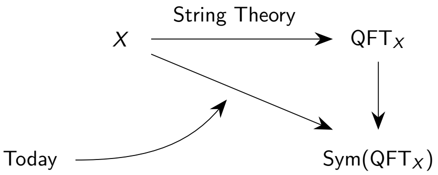
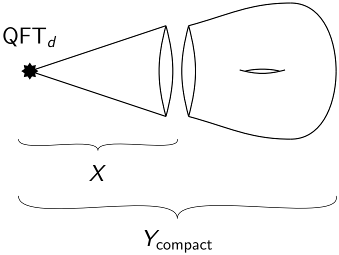


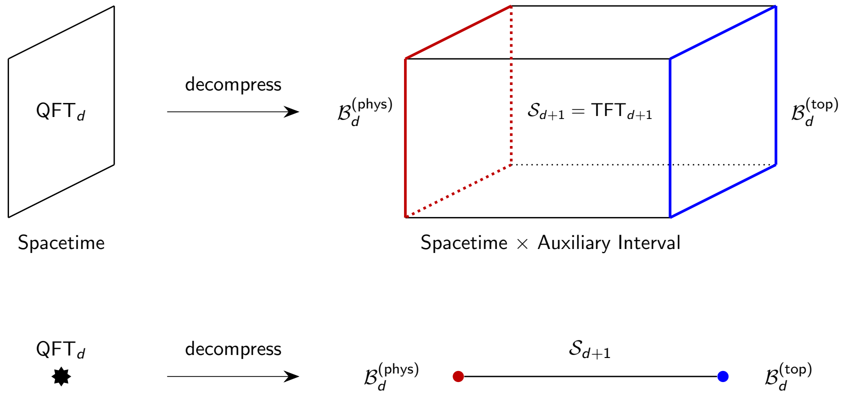
Motivation: Quantum Gravity

- No-Global-Symmetries Hypothesis: There are no global symmetries in theories of Quantum Gravity [Banks, Seiberg; 2011], [Harlow, Ooguri; 2018], [McNamara, Vafa; 2019], [Heckman, Hübner, Murdia; 2024], ...
- String Theory → UV completion of QFT_X to a theory of Quantum Gravity by local model construction $X \subset Y_{\text{compact}}$ [Cvetič, Heckman, Hübner, Torres; 2023]

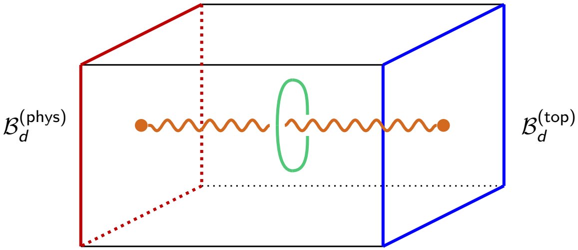


Field Theories and their Symmetry Theories

- Q: What structures govern symmetries in QFT? (Discrete Symmetries) [Aharony, Seiberg, Tachikawa; 2013], ...
- A: Symmetries of a $\mathrm{QFT}_d \to \mathrm{TFT}_{d+1}$ + Boundary Conditions [Gaiotto, Kulp; 2021], [Apruzzi, Bonetti, García Etxebarria, Hosseini, Schäfer-Nameki; 2021], [Freed, Moore, Teleman; 2022]
- Benefits: 1) separate the topological symmetry data of the QFT from non-topological data, 2) describe the QFT and all its gaugings simultaneously



The auxiliary interval is completely auxiliary.

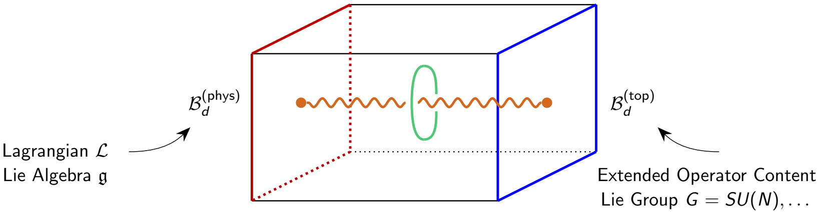


Spacetime $\times$ Auxilary Interval

Green: Symmetry Operator

Brown: Defect Operator

Example: (S)YM Theory in dimension $D = 4$



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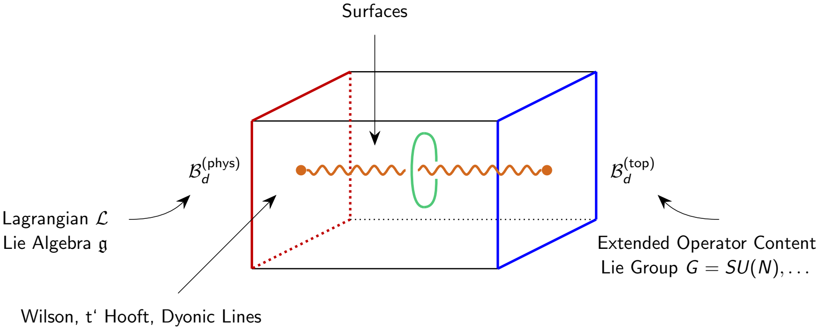

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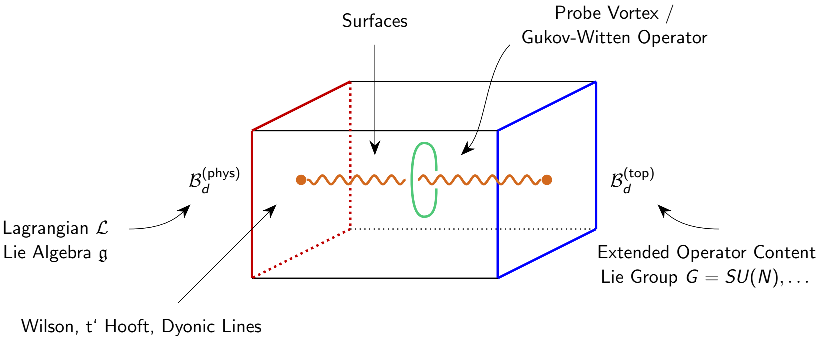

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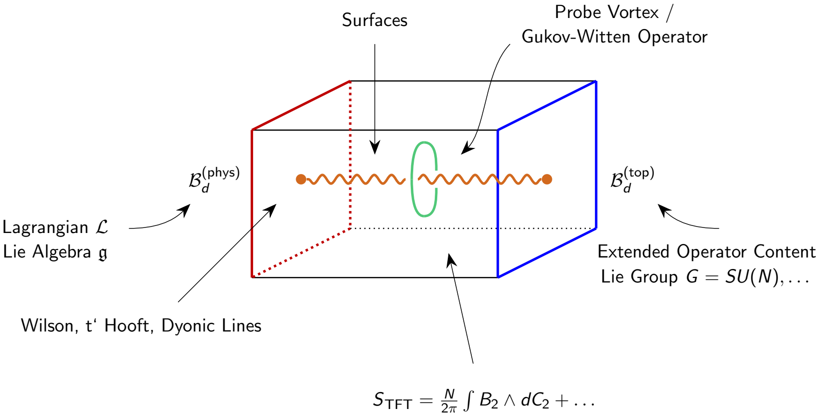

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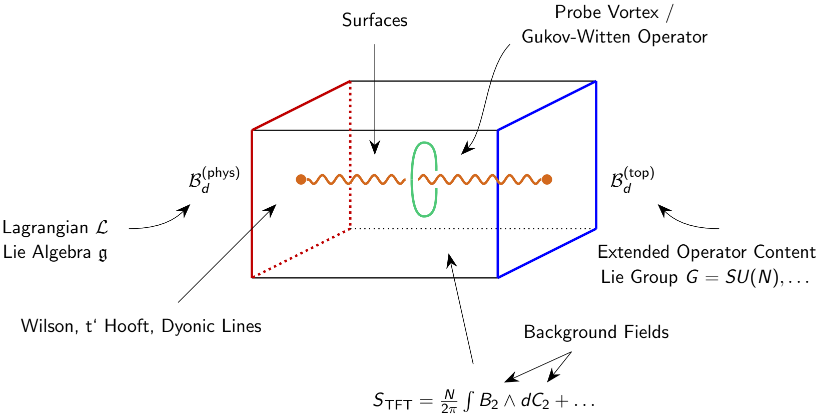

Generalizations: SymTrees and Cheesesteaks

Set of Data to be specified:

- Extra-dimensional support $\leftrightarrow$ manifold with boundary
- Field Content $\leftrightarrow$ Symmetries of the QFT
- TFT Action $\leftrightarrow$ Quantum Dual Symmetries, Anomalies
- Boundary Conditions $\leftrightarrow$ Edgemodes + Global Form

How is this data mapped under RG flow / limits in moduli space?
How to incorporate non-discrete symmetries?

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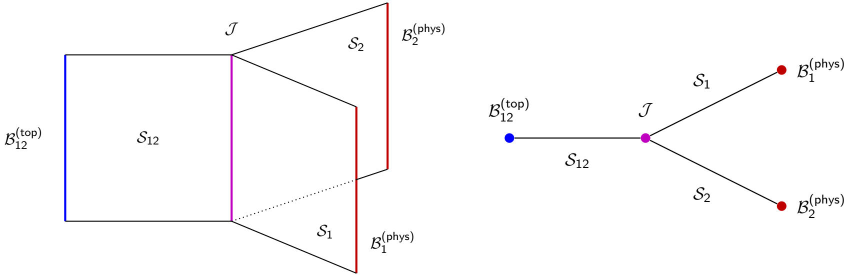
Generalizations: SymTrees

Limits decoupling modes → QFT splits into multiple sectors

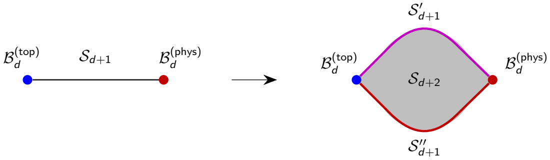
Each sector has its own symmetry theory, combining to the overall symmetry theory via junction associated with the decoupled data

[Baume, Heckman, Hübner, Torres, Turner, Yu; 2023]



Generalizations: Closed Cheesesteaks

If we can decompress a QFT into a system in one dimension higher, can we iterate this operation? [Cvetič, Donagi, Heckman, Hübner, Torres; 2024]

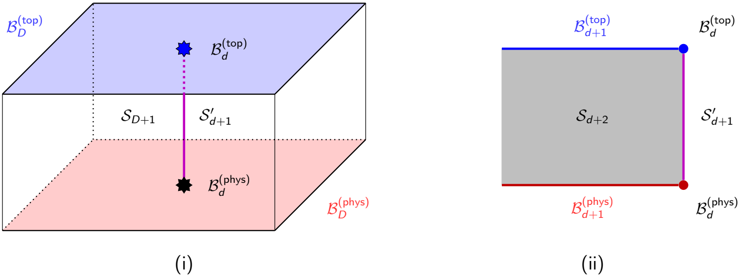


Manifold with Boundary $\rightarrow$ Manifold with Corners

[Cordova, Holfester, Ohmori; 2024], [Cvetič, Donagi, Heckman, Hübner, Torres; 2024], [García Etxebarria, Huertas, Uranga; 2024], [Bhardwaj, Pajer, Schäfer-Nameki; 2024], [Choi, Rayhaun, Zheng; 2024]

Generalizations: Open Cheesesteaks

Very natural for given $\mathrm{QFT}_d \subset \mathrm{QFT}_D$: [Cvetič, Donagi, Heckman, Hübner, Torres; 2024]

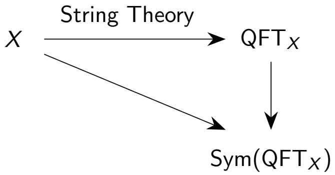


$(i) \to (ii)$: Reorganizing via compactification $\mathcal{B}_D^{(\text{phys})} \to \mathcal{B}_{d+1}^{(\text{phys})}$

Geometric Engineering

Punchline: All these extra-dimensional structures arise naturally in the Geometric Engineering of QFTs

Setup: non-compact, singular space X (purely geometric background)



$X = \mathbb{C}^n/\Gamma$, Elliptic CYs (eg. NHCs), G_2-Orbifolds, ...

QFT_X = Gauge Theories, SCFTs, Non-Supersymmetric QFTs, ...

Geometric Engineering

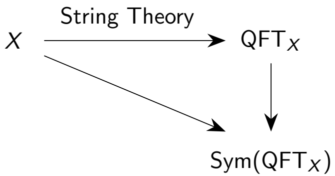
Organization: Singular Locus $\mathscr{S} \subset X$

$$\mathscr{S}_0 \subset \mathscr{S}_1 \subset \dots \mathscr{S}_l \subset X$$

$\rightarrow$ # **Connected Components + Depth** l

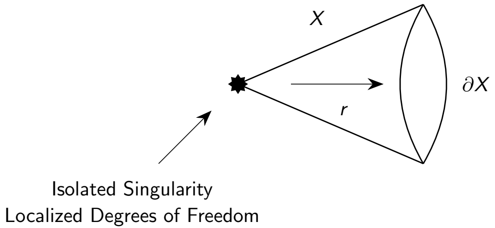
Example: $X = \mathbb{C}^2/\mathbb{Z}_N(1,-1)$, $\widetilde{\mathbb{C}^2/\mathbb{Z}_N}(1,-1)$, $\mathbb{C}^3/\mathbb{Z}_{2n}(1,1,-2)$

Setup: non-compact, singular space X (purely geometric background)



Case 0: Isolated Singularities

Sketch of Geometry:



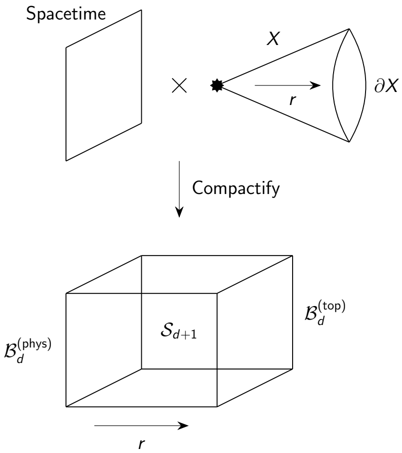
Example: $X = \mathbb{C}^2/\mathbb{Z}_N$

Example in M-theory: $\text{QFT}_X = $ 7D SYM

Example in IIB: $\text{QFT}_X = $ 6D (2,0) A_{N-1} SCFT

Case 0: Symmetry Theory

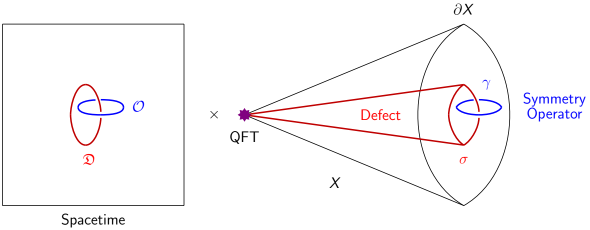
1) Compactify on constant radius slices [Apruzzi, Bonetti, García Etxebarria, Hosseini, Schäfer-Nameki; 2021]
2) Restrict to Topological Sector
3) Localized modes $\rightarrow \mathcal{B}_d^{\text{phys}}$
4) Supergravity BCs $\rightarrow \mathcal{B}_d^{\text{top}}$



Case 0: Symmetry and Defect Operators

Using Branes: [Del Zotto, Heckman, Park, Rudelius; 2015], [Morrison, Schäfer-Nameki, Willett; 2020], [Lakshya Bhardwaj, MH, Schäfer-Nameki; 2021], . . . , [García Etxebarria; 2022], [Apruzzi, Bah, Bonetti, Schäfer-Nameki; 2022], [Heckman, MH, Torres, Zhang; 2022], [Del Zotto, Heckman, Meynet, Moscrop, Zhang; 2022], . . .

Build Defect Operators (eg. Wilson, 't Hooft lines) and Symmetry Operators (eg. Gukov-Witten operators)



First goal achieved: Non-Lagrangian streamlined construction of topological operators

Key Observation

We fibered the Geometry X

$$\partial X\big|_r \hookrightarrow X \rightarrow \text{Interval}_r$$

and compactified on the fibers.

This was a **local** construction, i.e., we can apply it to **any fibration**

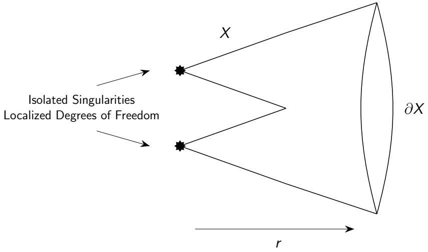
$$F \hookrightarrow X \rightarrow B$$

to produce a symmetry theory on the base B.

[Baume, Heckman, Hübner, Torres, Turner, Yu; 2023], [Cvetič, Donagi, Heckman, Hübner, Torres; 2024]

Case 1: Multiple Isolated Singularities

Sketch of Geometry:



Example: $X = \widetilde{\mathbb{C}^2/\mathbb{Z}_N}$

Example in M-theory: $\mathrm{QFT}_X =$ Higgsed 7D SYM

Example in IIB: $\mathrm{QFT}_X =$ 6D (2,0) A_{N-1} SCFT on CB

Case 1: SymTrees = SymTFTs + Junctions

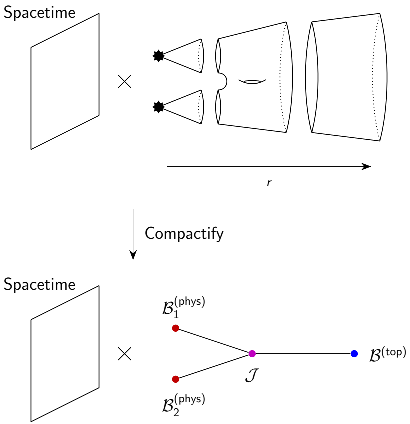
1) Compactify on constant radius slices, possibly disconnected

[Baume, Heckman, Hübner, Torres, Turner, Yu; 2023]

2) Restrict to Topological Sector

3) Localized modes $\rightarrow \mathcal{B}_1^{(\text{phys})}, \mathcal{B}_2^{(\text{phys})}$

4) Supergravity BCs $\rightarrow \mathcal{B}^{(\text{top})}$



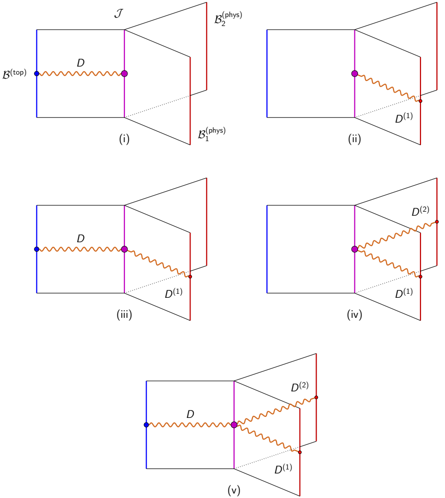
Case 1: Symmetry and Defect Operators

Again, using Branes

1) build Defect Operators

2) build Symmetry Operators

These now interact with junction $\mathcal{J}$



Case 2: Non-Isolated Singularities

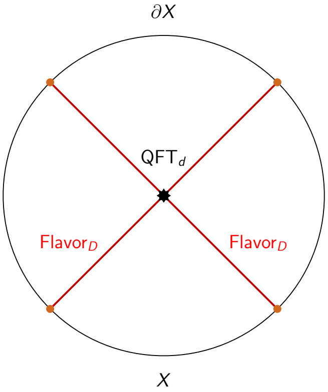
Sketch of Geometry:

"Intersecting Flavor Branes"

Example: $X = \mathbb{C}^3/\mathbb{Z}_{6n}(1,2,-3)$

M-theory: $\text{QFT}_X = $ 5D SCFT

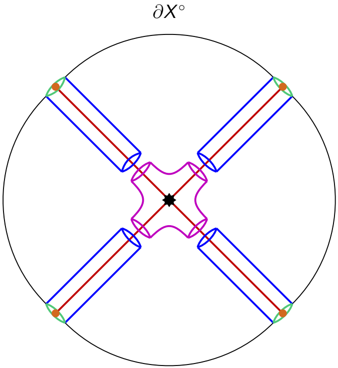
IIB: $\text{QFT}_X = $ 4D Theory of AD Type



Case 2: Cheesesteak = Symmetry TFTs + Corners

1) Consider generic Normal Geometry to the Flavor Brane

2) Construct the Symmetry Theory for the Flavor Brane



Case 2: Cheesesteak = Symmetry TFTs + Corners

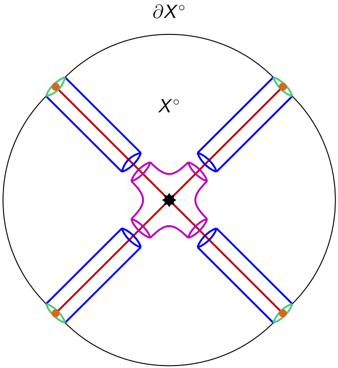
1) Consider generic Normal Geometry to the Flavor Brane

2) Construct the Symmetry Theory for the Flavor Brane

3) Consider exceptional Normal Geometry at singularity enhancement locus

4) Construct the Symmetry Theory for QFT_d

5) ... ...

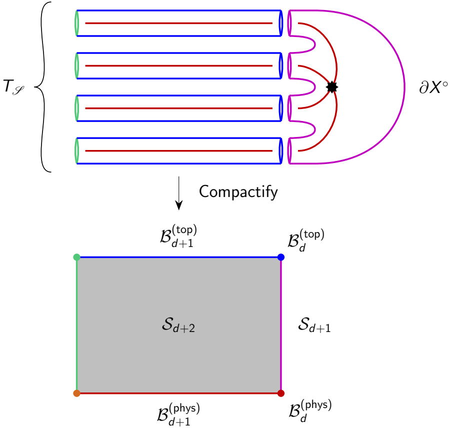


Geometry X and tubular nhood $T_{\mathscr{S}}$ are homotopic

$\Rightarrow$ Non-flat fibration of X:

$$F \hookrightarrow X \rightarrow \text{Square}$$

[Cvetič, Donagi, Heckman, Hübner, Torres; 2024]



QFT_d is a corner mode to $\mathcal{S}_{d+2}$

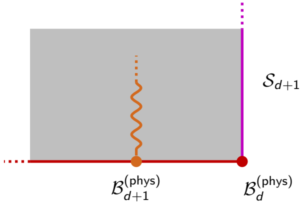
QFT_d is an end of the world mode to $\mathcal{B}_{d+1}^{(\text{phys})}$

$\mathcal{S}_{d+1}$ is a boundary mode to $\mathcal{S}_{d+2}$

Case 2: Symmetry and Defect Operators

Example, 2-group Symmetries: [Cvetič, Heckman, Hübner, Torres; 2022], [Del Zotto, García Etxebarria,Schäfer-Nameki; 2022], [Cvetič, Donagi, Heckman, Hübner, Torres; 2024]

Push Defect Operator of $\mathcal{B}^{(\text{phys})}_{d+1}$ into $\mathcal{B}^{(\text{phys})}_{d}$



For example, flavor lines W_F can decompose into # color lines W_C

$$W_F = W_C^{\#}$$

Case 3: Generalizations

1) The general case with singular locus $\mathscr{S} \subset X$

$$\mathscr{S}_0 \subset \mathscr{S}_1 \subset \dots \mathscr{S}_I \subset X$$

now follow from combining case 1 and 2.

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now follow from combining case 1 and 2.

2) "Purely Geometric Backgrounds" are a non-physical notion. Duality relates geometric backgrounds to non-geometric backgrounds.

$\Rightarrow$ Starting point for extending to more general backgrounds

Application: ~~Symmetries~~ in Quantum Gravity

What if X models some local patch of a compact space Y?

Compact $Y \Rightarrow$ String / M-theory on Y gives a theory of quantum gravity

No symmetries of QFT_X can remain

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Example, singular K3 surfaces: $Y = T^4/\mathbb{Z}_2$ [Cvetič, Heckman, Hübner, Torres; 2023]

Contains $2^4 = 16$ patches modelled on $X = \mathbb{C}^2/\mathbb{Z}_2$

Gluing patches to Y completes $\text{QFT}_{\mathbb{C}^2/\mathbb{Z}_2}^{\otimes 16}$ to a theory of quantum gravity

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Gluing $\Rightarrow$ Adding States, Breaking Symmetries, Gauging Symmetries

Application: ~~Symmetries~~ in Quantum Gravity

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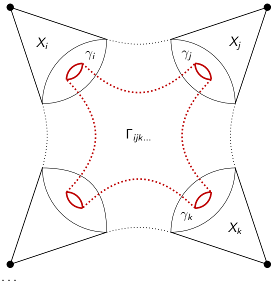
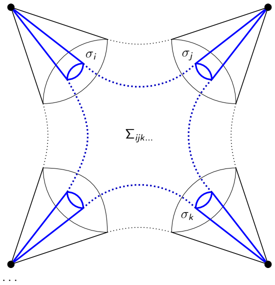


(1) : Symmetry Operators



(2) : Defect Operators

Invertible symmetries: Formalized via Mayer-Vietoris long exact sequence

[Cvetič, Heckman, Hübner, Torres; 2023]

Application: ~~SUSY~~ String Constructions

In examples so far X had special holonomy

$\Rightarrow$ covariantly constant spinors

$\Rightarrow$ supersymmetry

However, discrete symmetries are topological features and do not require holormophy to be extracted from X

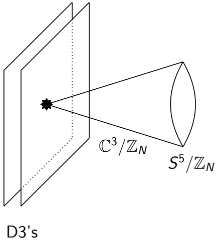
$\Rightarrow$ susy helped us understand **which theory** we were discussing

Now, consider supersymmetry-breaking backgrounds X

$\Rightarrow$ conversely, topological considerations constrain the resulting theory

Application: ~~SUSY~~ String Constructions

Example: D3-brane probes of $X = \mathbb{C}^3/\mathbb{Z}_N$ with $\mathbb{Z}_N \not\subset SU(3)$ [Chakrabhavi, Braeger, Heckman, Hübner; 2024], [Heckman, Hübner; 2024], [Chakrabhavi, Braeger, Heckman, Hübner; WIP]



Small 't Hooft Coupling: Tachyon in closed string sector $\rightarrow$ Instability

[Morrison, Narayan, Plesser; 2004], [Dymarsky, Klebanov, Roiban; 2005]

Large 't Hooft Coupling: Bubble of Nothing in AdS dual $\rightarrow$ Instability'

[Horowitz, Orgera, Polchinski; 2008]

Application: ~~SUSY~~ String Constructions

Example: D3-brane probes of $X = \mathbb{C}^3/\mathbb{Z}_N$ with $\mathbb{Z}_N \not\subset SU(3)$

Instability is a function of 't Hooft coupling

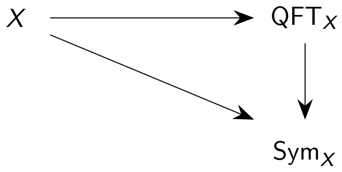
"Intermediate value theorem": Evidence for transition between instability types, at some finite 't Hooft coupling

Celestial Considerations: Evidence for a non-susy D3-brane CFT

[Heckman, Hübner; 2024]

Summary (Framework)

- We studied the geometric engineering map $X \to \mathrm{Sym}_X$



  for singular geometric backgrounds X.

- The support of Sym_X was a CW-complex with edge / corner modes; some topological, some physical.
- In particular, we gave a perspective on non-Abelian symmetries in the SymTFT framework.

Summary and Outlook

- We applied the mapping $X \mapsto \mathrm{Sym}_X$ to study

  a) Decoupling limits in String Theory

  b) Symmetry Inheritance for defects $\mathrm{QFT}_d \subset \mathrm{QFT}_D$

  c) No-Global-Symmetries Conjecture in QG

  d) Non-supersymmetric string constructions

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- We applied the mapping $X \mapsto \mathrm{Sym}_X$ to study
  a) Decoupling limits in String Theory
  b) Symmetry Inheritance for defects $\mathrm{QFT}_d \subset \mathrm{QFT}_D$
  c) No-Global-Symmetries Conjecture in QG
  d) Non-supersymmetric string constructions
- Future directions could inculde
  a) Brane systems, Class S (irregular punctures), . . .
  b) Geometrization of phase diagramms
  c) Representation Theory for Sym_X