Symmetries from Geometry

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2408.12600, 2406.08485, 2404.17639, 2401.09538, 2310.12980, 2307.13027, 2305.09665, 2304.03300, 2212.09743, 2209.03343 and WIP

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Motivation

- String/M-/F-theory:

SUGRA Singularity $\rightarrow$ QFT

Examples:

6d $\mathcal{N} = (2,0)$ SCFTs, 6d $\mathcal{N} = (1,0)$ SCFTs, 5d $\mathcal{N} = 1$ SCFTs, 4d Class S Theories, SUSY Gauge Theories, ..., YM Theory (?)

- However, there is No Free Lunch:

The above mapping is at least as complicated as the QFT.

Motivation

- Advantages of Characterizing QFTs via Singularities:
  1. Non-Lagrangian

     E.g.: Higgs Bundles, Orbifolds, Brane Systems, ...
  2. Efficient Parametrization

     E.g.: Phases (Confining Transition w/ G_2 Bryant-Salamon) ...
  3. Versatile

     E.g.: Duality Relations, UV completions, ...
- Singularity data is filtered:

  Topology, Differential Data, Special Structures, Metric Data, ...

Motivation

- Simplification: Focus on Topological Features of the QFT

$$\text{SUGRA Singularity} \rightarrow \text{QFT}|_{\text{top.}}$$

  E.g., Topological Operators, Anomalies, ...

- Noether's Theorem: [Noether, 1918]

$$\text{Symmetry} \;\rightarrow\; \text{Topological Operator}$$

- Modern Perspective: [Gaiotto, Kapustin, Seiberg, Willett; 2014]

$$\text{Symmetry} \;\leftarrow\; \text{Topological Operator}$$

Goal and Punchline

Q: Given a QFT_d constructed via a singular M-/F-/IIA/IIB background what are its topological operators (i.e., symmetries) and what are their properties (anomalies, fusions, associators, gauging relations, ...)?

A: Topological operators are constructed from branes and organized by a TFT_D in higher dimension $D > d$.

Remark: In this talk, the background will always take the form

$$\mathbb{R}^{\#} \times X$$

with internal dimensions X and flat spacetime $\mathbb{R}^{\#}$.

An Example

SCFT$_D$ in dimension $D = 5, 6$ are strongly coupled and non-Lagrangian, easily constructed in string theory, and progenitor theories for many lower-dimensional QFTs

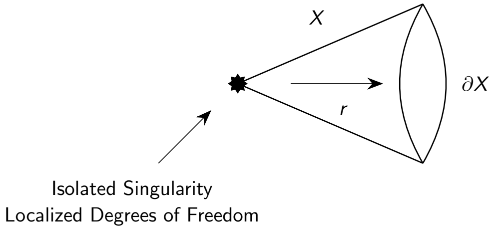
[Heckman, Morrison, Vafa; 2014], [Heckman, Morrison, Rudelius, Vafa; 2015], [Apruzzi, Lawrie, Lin, Schäfer-Nameki, Wang; 2019], [Closset, Del Zotto; 2020], ...

E.G.: M-theory on $\mathbb{C}^3/\mathbb{Z}_N = \text{Cone}(S^5/\mathbb{Z}_N)$ with isolated singularity $\rightarrow$ 5D SCFT which is an edge mode to

$$\text{TFT}_6 = \int_{\text{Spacetime}\times I} \frac{N}{2\pi} B_2 \wedge dB_3 + \eta_{S^5/\mathbb{Z}_N} B_2 \wedge B_2 \wedge B_2 + \ldots$$

where B_2, B_3 are 1-,2-form $\mathbb{Z}_N$ symmetry background fields respectively and anomaly $\eta_{S^5/\mathbb{Z}_N} \in \mathbb{Q}$.

Sketch of Geometry:



Example: $X = \mathbb{C}^3/\mathbb{Z}_N$

Example in M-theory: $\text{QFT}_X = \text{5D SCFT}$

Example in IIB: $\text{QFT}_X = \text{4D SCFT}$

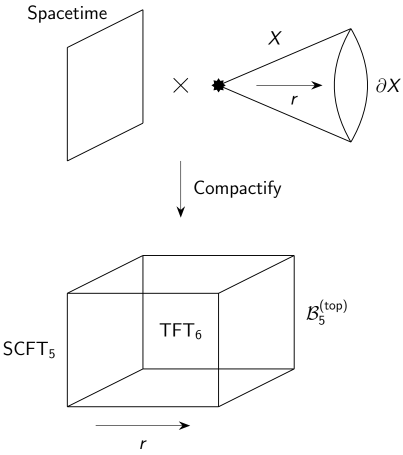
Geometry $\rightarrow$ Symmetries

1) Compactify on const. radius slices
[Apruzzi, Bonetti, García Etxebarria, Hosseini, Schäfer-Nameki; 2021]

2) Restrict to Topological Sector

3) Localized modes $\rightarrow \mathcal{B}_5^{\text{(phys)}} = \text{SCFT}_5$

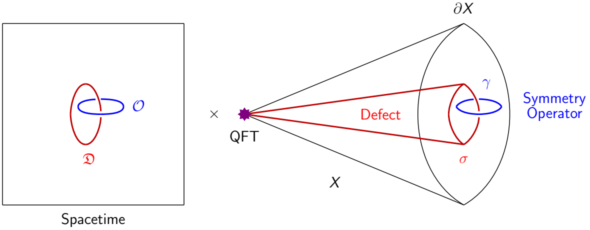
4) Supergravity BCs $\rightarrow \mathcal{B}_5^{\text{(top)}}$



Geometry → Symmetry and Defect Operators

Using Branes: [Del Zotto, Heckman, Park, Rudelius; 2015], [Morrison, Schäfer-Nameki, Willett; 2020], [Lakshya Bhardwaj, MH, Schäfer-Nameki; 2021], ..., [García Etxebarria; 2022], [Apruzzi, Bah, Bonetti, Schäfer-Nameki; 2022], [Heckman, MH, Torres, Zhang; 2022], [Del Zotto, Heckman, Meynet, Moscrop, Zhang; 2022], ...

Build Defect Operators (eg. Wilson, 't Hooft lines) and Symmetry Operators (eg. Gukov-Witten operators)



Key Observation

We fibered the Geometry X

$$\partial X\Big|_r \hookrightarrow X \rightarrow \text{Interval}_r$$

and compactified on the fibers.

This was a **local** construction, i.e., we can apply it to **any fibration**

$$F \hookrightarrow X \rightarrow B$$

to produce a symmetry theory on the base B.

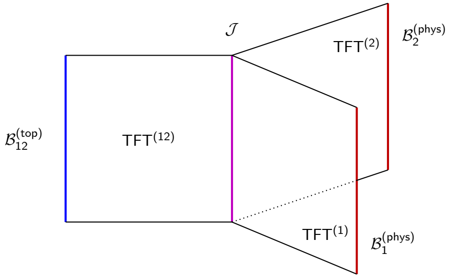
[Baume, Heckman, Hübner, Torres, Turner, Yu; 2023], [Cvetič, Donagi, Heckman, Hübner, Torres; 2024]

Generalizations: SymTrees

Limits decoupling modes $\rightarrow$ QFT splits into multiple sectors

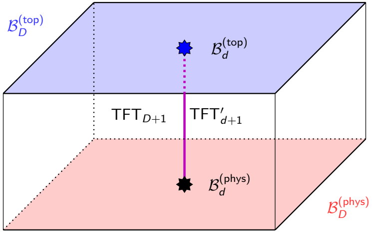
Each sector has its own symmetry theory, combining to the overall symmetry theory via junction associated with the decoupled data

[Baume, Heckman, Hübner, Torres, Turner, Yu; 2023]

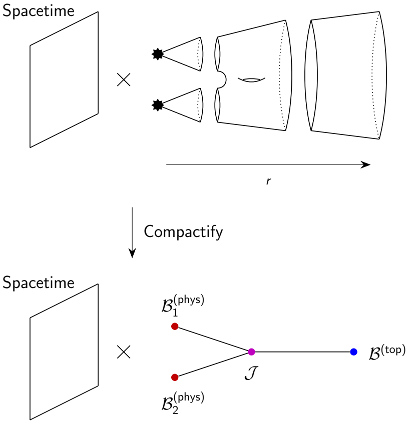


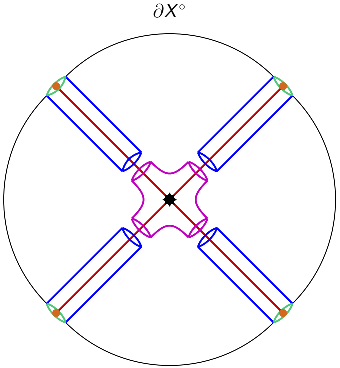
Generalizations: Cheesesteaks

Field Theory Systems: $\mathrm{QFT}_d \subset \mathrm{QFT}_D$: [Cvetič, Donagi, Heckman, Hübner, Torres; 2024]



Geometry → SymTrees and Cheesesteaks

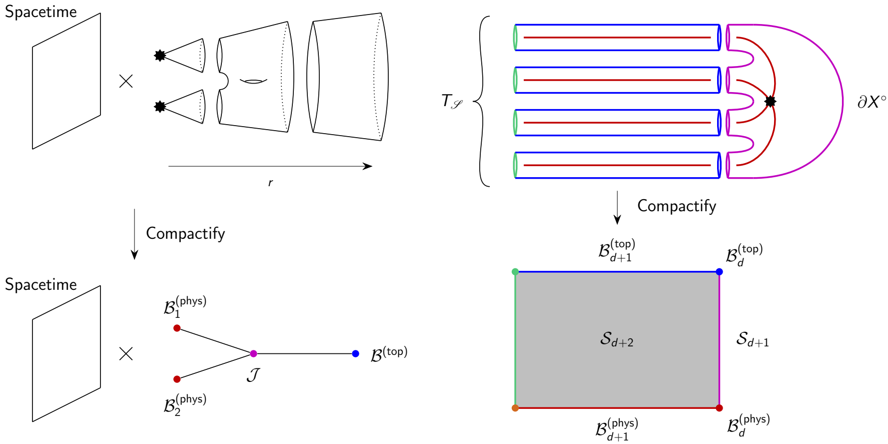




Left: 2 disjoint isolated singularities, Right: Non-Isolated Singularities

E.g.: Left: Higgsed Stack of Branes, Right: Intersecting Stacks of Branes

Geometry → SymTrees and Cheesesteaks



Left: 2 disjoint isolated singularities, Right: Non-Isolated Singularities

Outlook

Applications to

a) Decoupling limits in String Theory

b) Symmetry Inheritance for defects $\text{QFT}_d \subset \text{QFT}_D$

c) No-Global-Symmetries Conjecture in QG

d) Non-supersymmetric string constructions

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Thank you for your time and attention!